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A Novel Single Valued Neutrosophic Hesitant Fuzzy Time Series Model: Applications in Indonesian and Argentinian Stock Index Forecasting

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ABSTRACT This paper proposed a novel first-order single-valued neutrosophic hesitant fuzzy time series (SVNHFTS) forecasting model. Our aim is to improve the previously proposed neutrosophic time series (NTS) model by incorporating the degree of the hesitancy using single-valued neutrosophic hesitant fuzzy set (SVNHFS) model instead of single-valued neutrosophic set (SVNS). Our paper's novelty is that we incorporate an algorithm that automatically converts the crisp dataset into the neutrosophic set that eliminates the need for experts' input or opinions in determining the membership in each of the partitioned neutrosophic set. We also incorporate Markov Chain algorithm in the de-neutrosophication process to include the weightage of the repeating neutrosophic logical relationships (NLRs). Our paper's significant contribution is to add to the existing body of knowledge related to fuzzy time series (FTS) by developing a new FTS model based on SVNHFS, one of the improved version of fuzzy sets, since this area of research is still relatively underdeveloped. To determine our proposed model's capability, we apply our proposed SVNHFTS model to three real datasets while also comparing the result to the other FTS models based on improved versions of fuzzy sets. Our datasets include benchmark enrollment data of University of Alabama, IDX Composite (Indonesian composite stock index), and MERVAL index (Argentinian composite stock index). The result shows that our proposed SVNHFTS model outperforms most of the other FTS models in terms of AFE and RMSE, especially the previously proposed NTS model.

INDEX TERMS Single-valued neutrosophic hesitant fuzzy set (SVNHFS), single-valued neutrosophic hesitant fuzzy time series (SVNHFTS), neutrosophic time series (NTS), fuzzy time series (FTS).

I. INTRODUCTION

Fuzzy time series (FTS) has been quite a really important research topic in forecasting since its conception by Song & Chissom in 1993 [1], where all the time series data is converted from crisp set to linguistic variable (i.e. fuzzy set) in order to capture the uncertainty or "fuzziness" of the data movement. As conventional time series (regression, moving average, auto-regressive moving average, etc.) cannot capture the imprecise knowledge (i.e. non-probabilistic uncertainties and linguistic variables) within the dataset [2]–[4], fuzzy time series forecasting method can overcome these problems and increase the accuracy of the forecasted values. Fuzzy time series method is built based on fuzzy logic [5]–[7], where the universe of discourse is partitioned into several representative linguistic variables, fuzzification of the crisp time series data

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according to the partitioned universe of discourse, establishing rule bases (i.e. fuzzy logical relationships or FLRs) for defuzzification, and defuzzification of the fuzzified data into the crisp output (i.e. forecasted value) according to the established FLRs.

Since its conception approximately 25 years ago, fuzzy time series has inspired a lot of other researchers to take part to improve the method in order to increase the efficiency and the accuracy of the forecasted result. The majority of the researches are more focused on improving the steps of fuzzy time series method, which are optimizing the partition method and the length of the partition, with some of the most notable researches in [8]–[11] over the past 25 years and some of the newest ones are [12]–[14] in the past five years; and optimizing the rule base during fuzzification and defuzzification to get the forecasted values, with some of the most notable researches in [15]–[19] over the past 25 years and some of the newest ones are [20]–[23] in the past five years. Other researchers have also combined fuzzy time series with other forecasting method to improve the efficiency of the fuzzy time series, with some of the most notable researches in [24]–[28] over the past 25 years and some of the newest ones are [29]–[32] in the past five years. However, in almost all of these researches, they still utilized the original fuzzy set theory that was proposed by Zadeh in 1965 [7], where it can only assume one membership value for each members in the set. Meanwhile, the fuzzy set theory has also improved vastly since its conception, where the degree of the hesitancy and indeterminacy are introduced to better capture the uncertainty and vagueness of the membership function for each determined linguistic variable.

Over the past 50 years, a lot of the improved versions of fuzzy set theory have been developed to capture the indeterminacy, such as intuitionistic fuzzy set (IFS) by Atanassov in 1986 [33], neutrosophic set (NS) by Smarandache in 1995 [34], and single-valued neutrosophic set (SVNS) by Wang, *et al.* in 2012 [35]; and to capture the hesitancy, such as probabilistic fuzzy set (PFS) by Liu & Li in 2005 [36], hesitant fuzzy set (HFS) by Torra in 2010 [37], dual hesitant fuzzy set (DHFS) by Zhu, *et al.* in 2012 [38], and hesitant probabilistic fuzzy set (HPFS) by Zhou & Xu in 2017 [39]. Special mention goes to neutrosophic set, developed by Smarandache in 1995 [34] to generalize both interval-valued fuzzy set and intuitionistic fuzzy set with truth, falsity, and indeterminacy membership function, with the value of each can take a real standard or non-standard subsets of without any restriction. However, neutrosophic set was not easily applicable, which was why SVNS was developed by Wang, *et al.* in 2012 [35], so that each of the truth, falsity, and indeterminacy membership function only have a certain value between 0 to 1, which made it more applicable widely.

To our knowledge, the earliest fuzzy time series based on an improved version of the fuzzy set theory was proposed by Joshi & Kumar in 2012 [2], [3], where they proposed a fuzzy time series forecasting method based on intuitionistic fuzzy set. Since then, there have been quite a development in fuzzy time series based on improved versions of fuzzy set theory, such as Gangwar & Kumar in 2014 [4], where they proposed a fuzzy time series forecasting method based on intuitionistic fuzzy set utilizing CPDA (cumulative probability distribution approach) partition method; Kumar & Gangwar in 2015 [40], where they proposed a fuzzy time series forecasting method induced by intuitionistic fuzzy sets; Bisht & Kumar in 2016 [41], where they proposed a fuzzy time series forecasting method based on hesitant fuzzy set utilizing triangular and CPDA partition method; Joshi *et al.* in 2016 [42], where they introduced intuitionistic fuzzy time series forecasting method; and improved by Kumar & Gangwar in the same year [43]; Bisht, *et al.* in 2017 [44], where they proposed a fuzzy time series forecasting method based on hesitant fuzzy set utilizing triangular and Gaussian partition method; Bisht, *et al.* in 2018 [45], where they proposed a fuzzy time series forecasting method integrating intuitionistic fuzzy set and dual hesitant fuzzy set; Gupta & Kumar in 2018 [46], where they proposed a fuzzy time series forecasting method based on hesitant probabilistic fuzzy set; and Gupta & Kumar in 2019 [47], where they proposed a fuzzy time series forecasting method based on probabilistic fuzzy set. In 2019, Abdel-Basset, *et al.* [48] introduced a fuzzy time series forecasting method based on single-valued neutrosophic set, or known as neutrosophic time series (NTS). In the same year, however, Singh and Huang [49] also proposed their version of NTS, with different algorithm, since they combined it together with quantum optimization algorithm. Singh also proposed his version of NTS combined with particle swarm optimization (PSO) in 2020 [50]. The accuracy of the forecasted result in each of these researches is much more improved compared to the conventional fuzzy time series using the original fuzzy set theory, indicating that using improved version of fuzzy set theory is beneficial in improving the fuzzy time series forecasting method.

In 2015, Ye [51] introduced single-valued neutrosophic hesitant fuzzy set (SVNHFS), where it incorporates hesitant attributes of hesitant fuzzy set with single-valued neutrosophic set. so that each of the truth, falsity, and indeterminacy membership function can have more than one possible value. This special type of fuzzy set incorporates both degrees of hesitancy and indeterminacy such that more of the vagueness and "fuzziness" attribute can be caught better. With the recent proposed neutrosophic time series by Abdel-Basset, *et al.* and Singh and Huang in 2019 [48-49] that shows a significant improvement in terms of the simplicity and the accuracy utilizing SVNS, where only the degree of the indeterminacy is caught, we are inspired to utilize SVNHFS to propose a novel fuzzy time series forecasting model, called single-valued neutrosophic hesitant fuzzy time series (SVNHFTS). Our aim is to improve the previously proposed NTS model by incorporating the degree of the hesitancy using SVNHFS instead of SVNS. With this paper, we look to narrow down the currently limited research of utilizing improved version of fuzzy sets to improve FTS model.

The rest of the sections are as follows. The brief theories of fuzzy set, HFS, NS, SVNHFS, FTS, and NTS are explained in Section 2. The proposed SVNHFTS forecasting model is explained in Section 3. The comparative study of the characteristic of the proposed SVNHFTS model to the other established FTS models utilizing improved versions of fuzzy set theory is explained in Section 4. After that, the application of the proposed SVNHFTS forecasting model using three different datasets with comparison to the compared models in Section 4 is presented in Section 5. Finally, we will conclude the paper and give our suggestions of improvement of the study in Section 6.

II. PRELIMINARIES

In this section, we will rehash several important concepts regarding fuzzy set, HFS, NS, and SVNHFS, while also noting down several important concepts from fuzzy time series and neutrosophic time series. More information regarding the concepts of FS, HFS, NS, SVNS, SVNHFS, fuzzy time series, and neutrosophic time series can be found in these articles [1], [7], [34], [35], [37], [48], [51], [52]. However, please take note that the definition of neutrosophic time series is referred from Abdel-Basset *et al.* [48], not from Singh and Huang [49].

Definition 2.1 [7]: Let X be the universe of discourse with each member denoted as x_i. So, if X has n members, then $X = \{x_1, x_2, \ldots, x_n\}$. A fuzzy set (FS) is a set A in X such that each member of A has a membership function $\mu_A(x)$ = a value between [0,1] which indicates the measure of the membership of each element in X. In mathematical form, there are two notations to describe A, which are

$$A = \{\langle x_1, \mu_A(x_1)\rangle, \langle x_2, \mu_A(x_2)\rangle, \ldots, \langle x_n, \mu_A(x_n)\rangle \,|x \in X\} \tag{1}$$

or

$$A = \frac{\mu_A(x_1)}{x_1} + \frac{\mu_A(x_2)}{x_2} + \frac{\mu_A(x_3)}{x_3} + \ldots + \frac{\mu_A(x_n)}{x_n} \tag{2}$$

Definition 2.2 [53]: Let t_1, t_2 and t_3 be real numbers with $t_1 < t_2 < t_3$. Triangular fuzzy number $T = (t_1, t_2, t_3)$ is a fuzzy number with membership function:

$$\mu_T(x) = m(x) = \begin{cases} \dfrac{x - t_1}{t_2 - t_1}, & x\epsilon[t_1, t_2] \\ \dfrac{t_3 - x}{t_3 - t_2}, & x\epsilon[t_2, t_3] \\ 0, & x < t_1 \text{ and } x > t_3 \end{cases} \tag{3}$$

Definition 2.3 [11]: Let s_1, s_2, s_3, and s_4 be real numbers with $s_1 < s_2 < s_3 < s_4$. Trapezoidal fuzzy number $S = (s_1, s_2, s_3, s_4)$ is a fuzzy number with membership function:

$$\mu_S(x) = m(x) = \begin{cases} \dfrac{x - s_1}{s_2 - s_3}, & x\epsilon[s_1, s_2] \\ 1, & x\epsilon[s_2, s_3] \\ \dfrac{s_4 - x}{s_4 - s_3}, & x\epsilon\,[s_3, s_4] \\ 0, & x < s_1 \text{ and } x > s_4 \end{cases} \tag{4}$$

Definition 2.4 [54]: Let m and σ^2 be the mean and the variance of the membership function of a fuzzy number. The membership function of Gaussian fuzzy number is

$$\mu_A(x) = e^{\frac{-(x-m)^2}{2\sigma^2}}, \quad \sigma \neq 0 \tag{5}$$

Definition 2.5 [37]: Let X be the universe of discourse with each members denoted as x_i. A HFS is a set A in X such that each member of A has a membership function μ_A, where μ_A must lie within the range [0,1], which indicates the possible values of the measure of the membership of each elements in A.

Definition 2.6 [34]: Let X be the universe of discourse with each member denoted as x_i. A NS is a set A in X such that each members of A has a truth-membership function $T_A(x)$, which indicates the possible values of the measure of the membership of each members in A; a indeterminacy-membership function $I_A(x)$, which indicates the possible values of the measure of the indeterminacy of each elements in A; and a falsity-membership function $F_A(x)$, which indicates the possible values of the measure of the non-membership of each elements in A. In mathematical notation,

$$A = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle \,|x \in X\} \tag{6}$$

where the value of $T_A(x)$, $I_A(x)$, and $F_A(x)$ are real standard or non-standard subsets of $]^-0, 1^+[$ without any restriction, and it has to be that $0 \leq \sup(T_A(x)) + \sup(I_A(x)) + \sup(F_A(x)) \leq 3$.

Definition 2.7 [35]: Let X be the universe of discourse with each member denoted as x_i. A SVNS is a set A in X such that each members of A has a truth-membership function $T_A(x)$, which indicates the measure of the membership of each elements in A; a indeterminacy-membership function $I_A(x)$, which indicates the measure of the indeterminacy of each elements in A; and a falsify-membership function $F_A(x)$, which indicates the measure of the non-membership of each elements in A. In mathematical notation,

$$A = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle \,|x \in X\} \tag{7}$$

where the value of each of $T_A(x)$, $I_A(x)$, and $F_A(x)$ is a singular real number between [0,1].

Definition 2.8 [51]: Let X be the universe of discourse with each member denoted as x_i. A SVNHFS is a set A in X such that each members of A has a truth-membership function $\tilde{T}_A(x)$, which indicates the degree of the membership of each elements in A; a indeterminacy-membership function $\tilde{I}_A(x)$, which indicates the degree of the indeterminacy of each elements in A; and a falsify-membership function $\tilde{F}_A(x)$, which signifies the measure of the non-membership of each elements in A. In mathematical notation,

$$A = \left\{\left\langle x, \tilde{T}_A(x), \tilde{I}_A(x), \tilde{F}_A(x)\right\rangle |x \in X\right\} \tag{8}$$

The condition for SVNHFS is that $0 \leq \delta, \gamma, \eta \leq 1$ and $0 \leq \gamma^+ + \delta^+ + \eta^+ \leq 3$, where $\gamma \in \tilde{T}_A(x), \delta \in \tilde{I}_A(x), \eta \in \tilde{F}_A(x), \gamma^+ = \bigcup_{\gamma \in \tilde{T}_A(x)} \max(\gamma)$, $\delta^+ = \bigcup_{\tilde{I}_A(x) \in \delta} \max(\delta)$, and $\delta^+ = \bigcup_{\eta \in \tilde{F}_A(x)} \max(\eta)$. $\tilde{M}_A(x) = \{\tilde{T}_A(x), \tilde{I}_A(x), \tilde{F}_A(x)$ is called a single valued neutrosophic hesitant fuzzy element (SVNHFE), with a simplified notation $\tilde{n} = \left\{\tilde{t}, \tilde{i}, \tilde{f}\right\}$.

Definition 2.9 [51]: Let $\tilde{n}_c$ and $\tilde{n}_d$ be two different SVNHFEs. The following operations of these two SVNHFEs are defined:

$$\begin{aligned}\tilde{n}_c \cup \tilde{n}_d &= \left\{\tilde{t} \in \left(\tilde{t}_c \cup \tilde{t}_d\right) | \tilde{t} \geq \max\left(\tilde{t}_c^-, \tilde{t}_d^-\right), \quad \tilde{i} \in \left(\tilde{i}_c \cap \tilde{i}_d\right) | \tilde{i} \right.\\ &\left. \leq \min\left(\tilde{i}_c^+, \tilde{i}_d^+\right), \tilde{f} \in \left(\tilde{f}_c \cap \tilde{f}_d\right) | \tilde{f} \leq \min\left(\tilde{f}_c^+, \tilde{f}_d^+\right)\right\} \quad (9)\end{aligned}$$

$$\begin{aligned}\tilde{n}_c \cap \tilde{n}_d &= \left\{\tilde{t} \in \left(\tilde{t}_c \cap \tilde{t}_d\right) | \tilde{t} \leq \min\left(\tilde{t}_c^+, \tilde{t}_d^+\right), \quad \tilde{i} \in \left(\tilde{i}_c \cup \tilde{i}_d\right) | \tilde{i} \right.\\ &\left. \geq \max\left(\tilde{i}_c^-, \tilde{i}_d^-\right), \tilde{f} \in \left(\tilde{f}_c \cup \tilde{f}_d\right) | \tilde{f} \geq \max\left(\tilde{f}_c^-, \tilde{f}_d^-\right)\right\} \quad (10)\end{aligned}$$

Definition 2.10 [51]: Let $\tilde{n}_r (r = 1, 2, \ldots, s)$ be an assortment of SVNHFEs. The aggregated result of single valued neutrosophic hesitant fuzzy weighted average operator (SVNHFWA) is classified as an SVNHFE, with the formula:

$$\begin{aligned}&SVNHFWA\left(\tilde{n}_1, \tilde{n}_2, \ldots, \tilde{n}_s\right) = \sum_{r=1}^{s} w_r \tilde{n}_r \\ &= \bigcup_{\gamma_1 \in \tilde{t}_1, \gamma_2 \in \tilde{t}_2, \ldots, \gamma_s \in \tilde{t}_s, \delta_1 \in \tilde{i}_1, \delta_2 \in \tilde{i}_2, \ldots, \delta_s \in \tilde{i}_s, \eta_1 \in \tilde{f}_1, \eta_2 \in \tilde{f}_2, \ldots, \eta_s \in \tilde{f}_s} \left\{ \times \left\{1 - \prod_{r=1}^{s}\left(1-\gamma_r\right)^{w_r}\right\}, \left\{\prod_{r=1}^{s} \delta_r^{w_r}\right\}, \left\{\prod_{r=1}^{s} \eta_r^{w_r}\right\}\right\} \quad (11)\end{aligned}$$

Definition 2.11 [51]: Let $\tilde{n}_r (r = 1, 2, \ldots, s)$ be an assortment of SVNHFEs. The cosine measure between $\tilde{n}_r (r = 1, 2, \ldots, s)$ and the ideal element $n^* = \langle 1, 0, 0 \rangle$ is:

$$\cos\left(\tilde{n}_r, n^*\right) = \frac{\frac{1}{l_r}\sum_{\gamma_i \in \tilde{t}_i} \gamma_i}{\sqrt{\left(\frac{1}{l_r}\sum_{\gamma_i \in \tilde{t}_i} \gamma_i\right)^2 + \left(\frac{1}{p_r}\sum_{\delta_i \in \tilde{i}_i} \delta_i\right)^2 + \left(\frac{1}{q_r}\sum_{\eta_i \in \tilde{f}_i} \eta_i\right)^2}} \quad (12)$$

where l_r, p_r, and q_r are the number of elements in $\tilde{t}_r, \tilde{i}_r, \tilde{f}_r$ for $r = 1, 2, \ldots, s$ respectively. Based on the cosine measure, two comparative laws can be constructed:

1) When the cosine measure of a SVNHFE is bigger than the cosine measure of another SVNHFE, the SVNHFE with bigger cosine measure is said to be superior to this other SVNHFE.
2) When the cosine measure of a SVNHFE has the same value as the cosine measure of another SVNHFE, these two SVNHFEs are said to be equivalent to each other.

Definition 2.12 [52]: Let $X(t)$ with $t = 0, 1, 2, \ldots$ be the universe of discourse where fuzzy set $f_i(t) (i = 1, 2, \ldots)$ are specified and $A(t)$ is a union of all $f_i(t)$. Then $A(t)$ is identified as a fuzzy time series defined on $X(t)$.

Definition 2.13 [52]: If only $A(t-1)$ cause $A(t)$, i.e. $A(t-1) \rightarrow A(t)$, this relationship can be explained as

$$A(t) = A(t-1) \circ R(t, t-1) \quad (13)$$

where $R(t, t-1)$ is the fuzzy relationship between $A(t-1)$ and $A(t)$. This equation is defined as the first-order fuzzy time series model of $A(t)$.

Definition 2.14 [52]: If at different t, $R(t, t-1)$ is independent of t, i.e. $R(t, t-1) = R(t-1, t-2)$, then $A(t)$ is defined as a time-invariant fuzzy time series. Otherwise, it is defined as time-variant fuzzy time series.

Definition 2.15 [48]: Let $Y(t)$ with $t = 0, 1, 2, \ldots$ be the universe of discourse where neutrosophic set $n_i(t) (i = 1, 2, \ldots)$ are specified and $B(t)$ is a union of all $n_i(t)$. Then $B(t)$ is identified as a neutrosophic time series defined on $Y(t)$.

Definition 2.16 [48]: If only $B(t-1)$ cause $B(t)$, i.e. $B(t-1) \rightarrow B(t)$, this relationship can be explained as

$$B(t) = B(t-1) \circ R'(t, t-1) \quad (14)$$

where $R'(t, t-1)$ is the neutrosophic relationship between $B(t-1)$ and $B(t)$. This equation is defined as the first-order neutrosophic time series model of $B(t)$.

Definition 2.17 [48]: If at different t, $\boldsymbol{R'(t, t-1)}$ is independent of t, i.e. $\boldsymbol{R'(t, t-1)} = \boldsymbol{R'(t-1, t-2)}$, then $\boldsymbol{B}(\mathbf{t})$ is defined as a time-invariant neutrosophic time series. Otherwise, it is defined as time-variant neutrosophic time series.

III. THE PROPOSED SVNHFTS FORECASTING MODEL

This section presents the proposed SVNHFTS model. Each step of the proposed model is explained next. However, please take note that in this paper, we only propose the first-order SVNHFTS model.

A. SINGLE VALUED NEUTROSOPHIC HESITANT FUZZY TIME SERIES (SVNHFTS)

Previously proposed neutrosophic time series by Abdel-Basset *et al.* in 2019 [48] has laid out the base framework of utilization fuzzy time series based on single-valued neutrosophic set. The relative simplicity of neutrosophic time series and the inclusion of the truth, falsity, and indeterminacy membership functions to incorporate the vagueness of the data movement has resulted in a more accurate calculation. However, the need of experts' input as the basis of the truth, falsity, and indeterminacy membership for each of the neutrosophic set from the partitioned universe of discourse may cause the calculation to be subjective and biased. Different opinions from different experts can cause the result from each expert to differ from each other, which can cause confusion as to what forecasted result shall be used. In order to avoid the subjectivity and biasness, we propose an algorithm that automatically converts the crisp dataset into neutrosophic set in terms of truth, falsity, and indeterminacy membership

TABLE 1. Base mapping table [9].

Range of the half of the average of the absolute difference	Basis
0.1 – 1.0	0.1
1.0 – 10.0	1
10.0 – 100.0	10
100.0 – 1000.0	100

functions, inspired by Abdel-Basset, *et al.*'s proposed neutrosophic set rule mining algorithm for big data analysis in 2018 [55]. Moreover, the degree of hesitancy is also included in the algorithm by utilizing several different partition methods to partition the universe of discourse. The aggregated result of all the possible values of the truth, falsity, and indeterminacy membership values from each of the generated neutrosophic set from each of the partition method shall be in single-valued neutrosophic hesitant fuzzy sets (SVNHFSs), which shall be used for generating neutrosophic logical relationships (NLRs) and the deneutrosophication process.

Moreover, the calculation rule of the forecasted values utilized by Abdel-Basset, *et al.* (2019) does not consider the repetition of the NLRs that happens more than once. In a relatively stationary data, if the repetition of the NLRs is not considered, it may cause quite a relatively big error in the forecasted value. To accommodate for this issue, we propose to take into account the number of repetition of NLRs that is observed, inspired by Tsaur's (2012) fuzzy time series forecasting method based on Markov Chain [19]. The overall forecasted result from these incorporations should be better in terms of the accuracy.

The steps of the proposed SVNHFTS are explained as follows:

Step 1: State the universe of discourse U with the upper and lower boundary are the maximum value D_{max} and the minimum value D_{min} of the historical data. The universe is then stated as $U = [D_{min} - D_1, D_{max} + D_2]$, where $D_1.D_2$ are arbitrary positive values chosen for convenience, usually to make the universe easier to be divided.

Step 2: To find the effective length of the partition needed to divide the universe of discourse, Huarng's (2001) method [9] will be used, as it is

- Compute the absolute difference from each successive data of D_{i-1} and D_i.
- Compute the average out of all the absolute differences obtained, and then divide the average by 2.
- Using the base mapping table from Table 1, the basis for the length of interval is determined:
- The basis is rounded up or down to the most convenient number to be used as the length of the partition to divide the universe of discourse, which is denoted as L.
- The number of the fuzzy numbers that have to be constructed is calculated with the following formula:

$$m = \frac{D_{max} + D_2 - (D_{min} - D_1)}{L} \tag{15}$$

The value m will be the basis of the number of the partitions.

Step 3: For the first partition method, triangular fuzzy number will be used as the basis of the partition:

- Divide the universe of discourse to m triangular fuzzy numbers according to the following sets:

$$\begin{aligned}
\tilde{N}_{a_1} &= [D_{min} - D_1, D_{min} - D_1 + L, D_{min} - D_1 + 2L] \\
\tilde{N}_{a_2} &= [D_{min} - D_1 + L, D_{min} - D_1 + 2L, D_{min} - D_1 + 3L] \\
&\vdots \qquad\qquad\qquad \vdots \\
\tilde{N}_{a_{m-1}} &= [D_{max} + D_2 - 2L, D_{max} + D_2 - L, D_{max} + D_2] \\
\tilde{N}_{a_m} &= [D_{max} + D_2 - L, D_{max} + D_2, D_{max} + D_2]
\end{aligned}$$

with truth, indeterminacy, and falsity membership function for each of the triangular fuzzy numbers (denoted $\tilde{N}_{a_i} = [D_a, D_b, D_c]$ generally for simplicity) inspired by the neutrosophication association rule mining algorithm [55] as below:

$$T_{\tilde{N}_{a_i}}(x) = \begin{cases} \dfrac{x - D_a}{L}, & D_a \le x < D_b \\ \dfrac{D_c - x}{L}, & D_b \le x \le D_c \\ 0, & \text{otherwise} \end{cases} \tag{16}$$

$$I_{\tilde{N}_{a_i}}(x) = \begin{cases} \dfrac{x - (D_a - 0.5L)}{L}, & D_a - 0.5L \le x < D_a + 0.5L \\ \dfrac{D_b - x}{0.5L}, & D_a + 0.5L \le x < D_b \\ \dfrac{x - D_b}{0.5L}, & D_b \le x < D_b + 0.5L \\ \dfrac{D_c + 0.5L - x}{L}, & D_b + 0.5L \le x \le D_c + 0.5L \\ 0, & \text{otherwise} \end{cases} \tag{17}$$

$$F_{\tilde{N}_{a_i}}(x) = \begin{cases} \dfrac{D_b - x}{L}, & D_a \le x < D_b \\ \dfrac{x - D_b}{L}, & D_b \le x \le D_c \\ 1, & \text{otherwise} \end{cases} \tag{18}$$

From Equation (16) to (18), calculate the truth, indeterminacy, and falsity membership value of each data.

- To cast hesitancy to the membership values computed from the triangular partition method, by using the same length of the partition for each triangular fuzzy number, m Gaussian fuzzy numbers are constructed, with μ equal to the midpoint of each triangular fuzzy numbers. However, since the Gaussian function is an exponential function and the value of the membership function can only be zero when $x \to \infty$, to approximate the standard deviation (σ) of Gaussian membership function which represents the width of the function, the following steps are used:

- Calculate the length of each of the triangular fuzzy numbers, since the constructed triangular fuzzy numbers is isosceles, it has to be that:

$$L_{\tilde{N}_{a_i}} = 2L, \quad i = 1, 2, 3, \ldots, m \tag{19}$$

- A constant denoted by n is used such that the standard deviation of each triangular fuzzy number is formulated as:

$$\sigma_{\tilde{N}_{b_i}} = \frac{2L}{n}, \quad i = 1, 2, 3, \ldots, m \tag{20}$$

- n needs to be calculated such that the truth membership value of the lower boundary and the upper boundary of the triangular fuzzy number is as close to 0 as possible. In this project, the value is set to 0.0001. This value is arbitrary and can be changed according to the preference of the decision maker and the dataset that is being used in the research. Therefore, for $i = 1, 2, 3, \ldots, m$:

$$\begin{aligned}
&\exp\left(-\frac{(x-\mu)^2}{2\left(\frac{2L}{n}\right)^2}\right) \\
&= 0.0001, \quad x = \mu \pm L \\
&\Rightarrow \exp\left(-\frac{(\mu \pm L - \mu)^2}{2\left(\frac{2L}{n}\right)^2}\right) = 0.0001 \\
&\Rightarrow exp\left(-\frac{L^2}{2\left(\frac{2L}{n}\right)^2}\right) = 0.0001 \\
&\Rightarrow exp\left(-\frac{n^2}{8}\right) = 0.0001 \\
&\Rightarrow -\frac{n^2}{8} = \ln(0.0001) \\
&\Rightarrow n = 4\left(\sqrt{2\ln(10)}\right)
\end{aligned} \tag{21}$$

- The Gaussian fuzzy numbers corresponding to the triangular fuzzy numbers can now be constructed with the following rule:

$$\begin{aligned}
\tilde{N}_{b_1} &= \left[D_{min} - D_1 + L, \frac{2L}{4\sqrt{2\ln(10)}}\right] \\
\tilde{N}_{b_2} &= \left[D_{min} - D_1 + 2L, \frac{2L}{4\sqrt{2\ln(10)}}\right] \\
&\vdots \qquad\qquad \vdots \\
\tilde{N}_{b_{m-1}} &= \left[D_{max} + D_2 - L, \frac{2L}{4\sqrt{2\ln(10)}}\right] \\
\tilde{N}_{b_m} &= \left[D_{max} + D_2, \frac{2L}{4\sqrt{2\ln(10)}}\right]
\end{aligned}$$

with truth and falsity membership function for each of the Gaussian fuzzy numbers (denoted $\tilde{N}_{b_i} = [\mu, \sigma]$ generally for simplicity) are as given below:

$$T_{\tilde{N}_{b_i}}(x) = \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right) \tag{22}$$

$$F_{\tilde{N}_{b_i}}(x) = 1 - \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right) \tag{23}$$

Constructing indeterminacy membership function of the Gaussian fuzzy numbers are slightly more complicated since the function is exponential, which means there is no true zero point unless $x \to \infty$. When the truth and falsity membership value is the same at 0.5, the value of the indeterminacy membership value should be the highest at 1, and the value of indeterminacy membership where truth membership value equals to 1 should be the lowest at 0, which is at the μ of each Gaussian fuzzy numbers. As the value of truth membership value approaches 1, the value of the indeterminacy membership should approach 0. Because of this, the function of the indeterminacy membership around the midpoint is a reverse Gaussian function, starting at 1 and ending on 0 at the midpoint. Therefore, to create indeterminacy membership function, the following steps will be used:

- Find the value of x_I that causes $T_{\tilde{N}_{b_i}}(x_I) = F_{\tilde{N}_{b_i}}(x_I) = 0.5$

$$\begin{aligned}
&\exp\left(-\frac{\left(x_{I_{\tilde{N}_{b_i}}} - \mu\right)^2}{2\left(\frac{2L}{4\sqrt{2\ln(10)}}\right)^2}\right) = 0.5 \\
&\Rightarrow \exp\left(-\frac{\left(x_{I_{\tilde{N}_{b_i}}} - \mu\right)^2}{\frac{L^2}{4\ln(10)}}\right) = 0.5 \\
&\Rightarrow 4\ln(10)\frac{\left(x_{I_{\tilde{N}_{b_i}}} - \mu\right)^2}{L^2} = -\ln(0.5) \\
&\Rightarrow \left(x_{I_{\tilde{N}_{b_i}}} - \mu\right)^2 = \frac{L^2\ln(2)}{4\ln(10)} \\
&\Rightarrow x_{I_{\tilde{N}_{b_i}}} = \mu \pm 0.5L\sqrt{\frac{\ln(2)}{\ln(10)}}, \quad i = 1, 2, 3, \ldots, m
\end{aligned} \tag{24}$$

For $i = 1, 2, 3, \ldots, m$, from $\mu - 0.5L\sqrt{\frac{\ln(2)}{\ln(10)}}$ to $\mu + 0.5L\sqrt{\frac{\ln(2)}{\ln(10)}}$, the indeterminacy membership function is a reverse Gaussian function with the membership value of 0 at μ. Setting membership value of 0.9999 at $\mu - 0.5L\sqrt{\frac{\ln(2)}{\ln(10)}}$ and $\mu + 0.5L\sqrt{\frac{\ln(2)}{\ln(10)}}$, we can use the value of n from Equation 3.7 to find the standard deviation of this reverse Gaussian membership function. Therefore,

$$L^*_{\tilde{N}_{b_i}} = \mu + 0.5L\sqrt{\frac{\ln(2)}{\ln(10)}} - \left(\mu - 0.5L\sqrt{\frac{\ln(2)}{\ln(10)}}\right)$$
$$\Rightarrow L^*_{\tilde{N}_{b_i}} = L\sqrt{\frac{\ln(2)}{\ln(10)}}, \quad i = 1, 2, 3, \ldots, m \tag{25}$$

$$\sigma^*_{\tilde{N}_{b_i}} = \frac{L^*_{\tilde{N}_{b_i}}}{n}$$
$$\Rightarrow \sigma^*_{\tilde{N}_{b_i}} = \frac{L\sqrt{\frac{\ln(2)}{\ln(10)}}}{4\sqrt{2\ln(10)}}$$
$$\Rightarrow \sigma^*_{\tilde{N}_{b_i}} = \frac{L}{4\ln(10)}\sqrt{\frac{\ln(2)}{2}}, \quad i = 1, 2, 3, \ldots, m \tag{26}$$

When $x < \mu - 0.5L\sqrt{\frac{\ln(2)}{\ln(10)}}$ or $x > \mu + 0.5L\sqrt{\frac{\ln(2)}{\ln(10)}}$, the indeterminacy membership function should become normal Gaussian function, however, since the maximum value is set at 0.9999, the overall indeterminacy membership function of the Gaussian fuzzy number for $i = 1, 2, 3, \ldots, m$ is:

$$I_{\tilde{N}_{b_i}}(x) = \begin{cases} 0.9999\exp\left(-\frac{\left(x-\left(\mu - 0.5L\sqrt{\frac{\ln(2)}{\ln(10)}}\right)\right)^2}{2\sigma^2_{\tilde{N}_{b_i}}}\right), & x < \mu - 0.5L\sqrt{\frac{\ln(2)}{\ln(10)}} \\ 1-\exp\left(-\frac{(x-\mu)^2}{2\left(\sigma^*_{\tilde{N}_{b_i}}\right)^2}\right), & \mu - 0.5L\sqrt{\frac{\ln(2)}{\ln(10)}} \le x < \mu + 0.5L\sqrt{\frac{\ln(2)}{\ln(10)}} \\ 0.9999\exp\left(-\frac{\left(x-\left(\mu + 0.5L\sqrt{\frac{\ln(2)}{\ln(10)}}\right)\right)^2}{2\sigma^2_{\tilde{N}_{b_i}}}\right), & x \ge \mu + 0.5L\sqrt{\frac{\ln(2)}{\ln(10)}} \end{cases} \tag{27}$$

Figure 1 depicts the truth, indeterminacy, and falsity membership function of Gaussian function better for better illustration of the proposed algorithm.
From Equation (22), (23), and (27), calculate the truth, indeterminacy, and falsity membership value of each data.

- Combine the membership value computed from the triangular fuzzy number and the Gaussian fuzzy number into a SVNHFS.

Step 4: For the second partition method, trapezoidal fuzzy number will be used as the basis of the partition [11]:

- Divide the universe of discourse to m trapezoidal fuzzy numbers according to the following sets:

$$\begin{aligned} \tilde{N}_{c_1} &= [D_{min} - D_1 - L, D_{min} - D_1, \\ &\qquad D_{min} - D_1 + L, D_{min} - D_1 + 2L] \\ \tilde{N}_{c_2} &= [D_{min} - D_1, D_{min} - D_1 + L, \\ &\qquad D_{min} - D_1 + 2L, D_{min} - D_1 + 3L] \\ &\vdots \qquad\qquad \vdots \\ \tilde{N}_{c_{m-1}} &= [D_{max} + D_2 - 3L, D_{max} + D_2 - 2L, \\ &\qquad D_{max} + D_2 - L, D_{max} + D_2] \\ \tilde{N}_{c_m} &= [D_{max} + D_2 - 2L, D_{max} + D_2 - L, \\ &\qquad D_{max} + D_2, D_{max} + D_2 + L] \end{aligned}$$

with truth, indeterminacy, and falsity membership function for each of the trapezoidal fuzzy numbers (denoted $\tilde{N}_{c_n} = [D_a, D_b, D_c, D_d]$ generally for simplicity) are inspired by neutrosophication association rule mining algorithm [55] as below:

$$T_{\tilde{N}_{c_n}}(x) = \begin{cases} \frac{x - D_a}{L}, & D_a \le x < D_b \\ 1, D_b \le x < D_c & \\ \frac{D_d - x}{L}, & D_c \le x \le D_d \\ 0, \text{otherwise} & \end{cases} \tag{28}$$

$$I_{\tilde{N}_{c_n}}(x) = \begin{cases} \frac{x - (D_a - 0.5L)}{L}, & D_a - 0.5L \le x < D_a + 0.5L \\ \frac{D_b + 0.5L - x}{L}, & D_a + 0.5L \le x < D_b + 0.5L \\ \frac{x - (D_b + 0.5L)}{L}, & D_b + 0.5L \le x < D_c + 0.5L \\ \frac{D_d + 0.5L - x}{L}, & D_c + 0.5L \le x \le D_d + 0.5L \\ 0, & \text{otherwise} \end{cases} \tag{29}$$

$$F_{\tilde{N}_{c_n}}(x) = \begin{cases} \frac{D_b - x}{L}, & D_a \le x < D_b \\ 0, & D_b \le x < D_c \\ \frac{x - D_c}{L}, & D_c \le x \le D_d \\ 1, \text{otherwise} & \end{cases} \tag{30}$$

From Equation (28) to (30), calculate the truth, indeterminacy, and falsity membership value of each data.

- To cast hesitancy to the membership values generation by the trapezoidal partition method, using the same length of the partition for each trapezoidal fuzzy number, m Gaussian fuzzy numbers are constructed with the same method as from the triangular partition method.

$$L_{\tilde{N}_{d_i}} = 3L, \quad i = 1, 2, 3, \ldots, m-1, m \tag{31}$$

$$\sigma_{\tilde{N}_{d_i}} = \frac{3L}{4\sqrt{2\ln(10)}}, \quad i = 1, 2, 3, \ldots, m-1, m \tag{32}$$

$$x_{I_{\tilde{N}_{d_i}}} = \mu \pm 0.75L\sqrt{\frac{\ln(2)}{\ln(10)}}, \quad i=1, 2, 3, \ldots, m-1, m \tag{33}$$

$$\sigma^*_{\tilde{N}_{d_i}} = \frac{3L}{8\ln(10)}\sqrt{\frac{\ln(2)}{2}}, \quad i = 1, 2, 3, \ldots, m-1, m \tag{34}$$

Therefore, the truth, indeterminacy, and falsity membership function of each of the Gaussian fuzzy numbers from trapezoidal partition method are:

$$T_{\tilde{N}_{d_i}}(x) = \exp\left(-\frac{(x-\mu)^2}{2\sigma^2_{\tilde{N}_{d_i}}}\right) \tag{35}$$

$$I_{\tilde{N}_{d_i}}(x) = \begin{cases} 0.9999\exp\left(-\frac{\left(x-\left(\mu-0.75L\sqrt{\frac{\ln(2)}{\ln(10)}}\right)\right)^2}{2\sigma^2_{\tilde{N}_{d_i}}}\right), & x < \mu - 0.75L\sqrt{\frac{\ln(2)}{\ln(10)}} \\ 1-\exp\left(-\frac{(x-\mu)^2}{2\left(\sigma^*_{\tilde{N}_{d_i}}\right)^2}\right), & \mu - 0.75L\sqrt{\frac{\ln(2)}{\ln(10)}} \le x < \mu + 0.75L\sqrt{\frac{\ln(2)}{\ln(10)}} \\ 0.9999\exp\left(-\frac{\left(x-\left(\mu+0.75L\sqrt{\frac{\ln(2)}{\ln(10)}}\right)\right)^2}{2\sigma^2_{\tilde{N}_{d_i}}}\right), & x \ge \mu + 0.75L\sqrt{\frac{\ln(2)}{\ln(10)}} \end{cases} \tag{36}$$

$$F_{\tilde{N}_{d_i}}(x) = 1-\exp\left(-\frac{(x-\mu)^2}{2\sigma^2_{\tilde{N}_{d_i}}}\right) \tag{37}$$

- Combine the membership value computed from the trapezoidal fuzzy number and the Gaussian fuzzy number into a SVNHFS.

Step 5: Calculate the weight of the triangular and the trapezoidal partition method by using Equation (38) and (39) below:

$$w_1 = \frac{L_1}{L_1 + L_2} \tag{38}$$

$$w_2 = \frac{L_2}{L_1 + L_2} \tag{39}$$

where:
w_1 = weight of triangular partition method
w_2 = weight of trapezoidal partition method
L_1 = length of each triangular fuzzy number
L_2 = length of each trapezoidal fuzzy number

Step 6: After the weight of each of the partition method is calculated, the weighted average of the midpoint of the triangular method and the trapezoidal method in the same order is calculated by using Equation (40). This weighted average of the midpoints will be used as the basis during neutrosophication in Step 7.

$$\text{Midpoint} = \left(w_1 \times \text{Midpoint}_{\text{triangular}}\right) + \left(w_2 \times \text{Midpoint}_{\text{trapezoidal}}\right) \tag{40}$$

Step 7: The neutrosophication of the data is done with the following rules:

- For each data, the two SVNHFSs for each order of fuzzy number generated from triangular and trapezoidal partition method are aggregated into one SVNHFS by using Equation (11). The weight value for each SVNHFS is already calculated from Equation (38) and (39).
- Calculate the cosine measure of each of the aggregated SVNHFS by using Equation (12).
- Rank all of the calculated cosine measures from each of the aggregated SVNHFS. The highest rank out of all the cosine measures from the order of the fuzzy numbers will be used as the neutrosophication of the data.

Step 8: Establish NLRs based on the order of the neutrosophication of successive data. If on time i, the SVNHFS is N_j and on time $i+1$, the SVNHFS is N_k, the established NLR is $N_j \to N_k$.

Step 9: Establish neutrosophic logical relationship groups (NLRGs) based on the constructed NLRs. Group the NLRs based on N_j.

Step 10: The forecasted values are calculated using two rules following [48], with slight modification inspired from [19]:

- **Rule 1**: If the neutrosophication value of data at time i is N_k and not caused by any other values, and the corresponding value according to NLRG cannot be found, then the forecasted value will equal —, i.e., no value.
- **Rule 2**: If the neutrosophication value of data at time i is N_k and caused by any N_j, check NLRG of this N_j, and
  - If NLRG of N_j is empty, the forecasted value is weighted midpoint average of N_j
  - If NLRG of N_j is one-to-one, the forecasted value is weighted midpoint average of N_k

TABLE 2. Methods of error measure to evaluate accuracy of predicted value from the proposed FTS [41].

No	Name	Formula	Acceptable Range	Equation No
1	RMSE (root mean square error)	$\sqrt{\frac{\sum_{i=1}^{n}(O_i - F_i)^2}{n}}$	---	(41)
2	R (correlation coefficient)	$\frac{n\sum O_iF_i - \sum O_i \sum F_i}{\sqrt{n(\sum O_i^2) - \sum O_i^2}\sqrt{n(\sum F_i^2) - \sum F_i^2}}$	$-1 \leq R \leq 1$	(42)
3	R^2 (coefficient of determination)	R^2	$0 \leq R^2 \leq 1$	(43)
4	Forecasting error (in percentage)	$\frac{\lvert F_i - O_i \rvert}{O_i} \times 100\%$	---	(44)
5	AFE (average forecast error) (in percentage)	$\frac{\text{sum of forecasting error}}{n}$	---	(45)
6	PP (performance parameter)	$1 - \frac{\text{RMSE}}{\sigma}$	$PP > 0$	(46)
7	MAD (mean absolute deviation)	$\frac{\sum_{i=1}^{n}\lvert F_i - O_i \rvert}{n}$	---	(47)
8	RSFE (running sum of forecasting error)	$\sum_{i=1}^{n}(F_i - O_i)$	---	(48)
9	TS (tracking signal)	$\frac{R_{sfe}}{M_{ad}}$	$-4 \leq TS \leq 4$	(49)

TABLE 3. List of other FTS models to be compared to the proposed SVNHFTS model.

Year	Name of authors	Reference	Model used	Utilized fuzzy set
2015	Kumar & Gangwar	[40]	FTS induced by IFS	IFS (intuitionistic fuzzy set)
2016	Bisht & Kumar	[41]	FTS based on HFS using triangular and CPDA (cumulative probability distribution approach) partition method	HFS (hesitant fuzzy set)
2016	Kumar & Gangwar	[43]	Intuitionistic FTS	IFS (intuitionistic fuzzy set)
2017	Bisht, et al.	[44]	FTS based on HFS using triangular and Gaussian membership function	HFS (hesitant fuzzy set)
2018	Bisht, et al.	[45]	FTS based on integration of DHFS and IFS	DHFS (dual hesitant fuzzy set) and IFS (intuitionistic fuzzy set)
2018	Gupta & Kumar	[46]	FTS based on HPFS	HPFS (hesitant probabilistic fuzzy set)
2019	Abdel-Basset, et al.	[48]	Neutrosophic time series	SVNS (single valued neutrosophic set)

- If NLRG of N_j is one-to-many, the forecasted value is the average of the weighted midpoint average of $N_{k1}, N_{k2}, \ldots, N_{kn}$. Note that the repeating NLRs are counted and weighted in the weighted average.

Step 11: Evaluate the accuracy of the forecasted value using all the formulas listed in Table 2.

IV. COMPARATIVE STUDIES OF CHARACTERISTICS OF SVNHFTS MODEL TO OTHER FTS MODELS

In this section, we apply our proposed SVNHFTS model in real datasets. Forecasting results obtained from the proposed model was compared with existing FTS models, [40], [41], [43]–[46], [48], listed in Table 3.

To know the inherent advantage of the proposed SVNHFTS model compared to the other models, the characteristics of the proposed SVNHFTS model is compared to the other models are listed below and summarized in Table 4 and Table 5:

- Compared to the other FTS models listed in Table 3, the proposed SVNHFTS model captures the most uncertainty and fuzziness of the data movement, which is depicted in terms of truth, indeterminacy, and falsity membership function, also with the hesitancy also depicted in terms of the possible values of truth, indeterminacy, and falsity membership function, even though not as complete as HPFS based FTS, where the probability of each possible value of the membership function is also taken into account during the fuzzification process.
- The proposed SVNHFTS model is already very much simplified in terms of defuzzification process, since the max-min operator is eliminated, reducing the workload and the complexity of the defuzzification process. In other compared FTS models other than NTS and SVNHFTS, the max-min operator is still used for defuzzification process, which means that there will be a lot of matrices construction for each FLR, slowing down the defuzzification process and rendering the models less efficient in terms of time and energy consumption.
- The proposed SVNHFTS model makes sure that there is always a forecasted value in each of the time interval, unlike some of the other compared models, such as Kumar & Gangwar's IFTS (intuitionistic fuzzy time series) [43] and Bisht, *et al.*'s DHFS based IFTS [45], where there can be a possibility of no available forecasted value during the defuzzification process.
- The proposed SVNHFTS model takes into account the weightage of repeated NLRs, unlike other compared FTS models, where the repeated NLRs are only listed once. This means that SVNHFTS may have higher accuracy when dealing with data with stagnant area but also with wider and more drastic movement.

In the next section, we apply our proposed SVNHFTS model using real datasets, and the result of the forecasted value from our proposed SVNHFTS model is compared to the other fuzzy time series models utilizing improved version of fuzzy sets.

V. APPLICATION OF SVNHFTS MODEL TO FORECAST REAL DATASETS

In this section, we apply our proposed SVNHFTS model using real datasets in order to test the performance of our model, and we compare the forecasted result to the other FTS models based on improved versions of fuzzy set. There are three different sets of data that are used for the application, which are enrollment data from University of

TABLE 4. Summary of comparison between the proposed SVNHFTS model and other established FTS models (Part 1).

Year	Name of authors	Reference	Model used	Utilized fuzzy set	Fuzziness	Falsity	Indeterminacy	Hesitancy
2015	Kumar & Gangwar	[40]	FTS induced by IFS	IFS (intuitionistic fuzzy set)	✓	✓	✓	×
2016	Bisht & Kumar	[41]	FTS based on HFS using triangular and CPDA (cumulative probability distribution approach) partition method	HFS (hesitant fuzzy set)	✓	×	×	✓
2016	Kumar & Gangwar	[43]	Intuitionistic FTS	IFS (intuitionistic fuzzy set)	✓	✓	✓	×
2017	Bisht, et al.	[44]	FTS based on HFS using triangular and Gaussian membership function	HFS (hesitant fuzzy set)	✓	×	×	✓
2018	Bisht, et al.	[45]	FTS based on integration of DHFS and IFS	DHFS (dual hesitant fuzzy set) and IFS (intuitionistic fuzzy set)	✓	✓	✓	✓
2018	Gupta & Kumar	[46]	FTS based on HPFS	HPFS (hesitant probabilistic fuzzy set)	✓	×	×	✓
2019	Abdel-Basset, et al.	[48]	Neutrosophic time series	SVNS (single valued neutrosophic set)	✓	✓	✓	×
2019	**Our proposed model**	**N/A**	**SVNHFTS**	**SVNHFS**	✓	✓	✓	✓

TABLE 5. Summary of comparison between the proposed SVNHFTS model and other established FTS models (Part 2).

Year	Name of authors	Reference	Model used	Utilized fuzzy set	Probability	Max-Min Operator	Guaranteed Forecasted Value	Weightage of FLR/NLR
2015	Kumar & Gangwar	[40]	FTS induced by IFS	IFS (intuitionistic fuzzy set)	×	✓	✓	×
2016	Bisht & Kumar	[41]	FTS based on HFS using triangular and CPDA (cumulative probability distribution approach) partition method	HFS (hesitant fuzzy set)	×	✓	✓	×
2016	Kumar & Gangwar	[43]	Intuitionistic FTS	IFS (intuitionistic fuzzy set)	×	✓	×	×
2017	Bisht, et al.	[44]	FTS based on HFS using triangular and Gaussian membership function	HFS (hesitant fuzzy set)	×	✓	✓	×
2018	Bisht, et al.	[45]	FTS based on integration of DHFS and IFS	DHFS (dual hesitant fuzzy set) and IFS (intuitionistic fuzzy set)	×	✓	×	×
2018	Gupta & Kumar	[46]	FTS based on HPFS	HPFS (hesitant probabilistic fuzzy set)	✓	✓	✓	×
2019	Abdel-Basset, et al.	[48]	Neutrosophic time series	SVNS (single valued neutrosophic set)	×	×	✓	×
2019	**Our proposed model**	**N/A**	**SVNHFTS**	**SVNHFS**	×	×	✓	✓

Alabama from 1971 to 1992 [56], IDX Composite (Indonesia's market index) yearly average of daily closing index from 1997 to 2018 [57], and weekly closing index of MERVAL(Argentina's market index) from February 15th, 2019 to September 20th, 2019 [58]. All the data points in the datasets are used to generate the NLRGs and the forecasted results' accuracy is checked for all the data points.

TABLE 6. Comparison of forecasted values of enrollment data of University of Alabama from 1971 to 1992 by using the proposed SVNHFTS model and other FTS models.

Year	Actual	Proposed Algorithm	[40]	[41]	[43]	[44]	[45]	[46]	[48]
1971	13055	---	---	---	---	---	---	---	---
1972	13563	13560	13500	13595.67	13693	13741	13753.86	13680.75	13650
1973	13867	13860	13500	13814.75	13693	13741	13744.65	13844.43	13950
1974	14696	14760	14500	14929.79	14867	14740.85	14567.78	14951.36	14850
1975	15460	15360	15500	15541.27	15287	15417.91	15498.37	15532.34	15450
1976	15311	15560	15500	15540.62	15376	15357.26	15411.79	15533.19	15450
1977	15603	15560	15500	15540.62	15376	15357.26	15498.37	15533.19	15450
1978	15861	15510	15500	15540.62	15376	15357.26	15498.37	15533.19	15450
1979	16807	16860	17000	16254.50	16523	16854.26	17296.95	16298.77	16950
1980	16919	17060	17000	17040.41	16606	17193.42	17452.72	17113.79	17150
1981	16388	17060	17000	17040.41	17519	17193.42	---	17113.79	17150
1982	15433	15360	15500	16254.50	16606	15441.40	15411.79	16298.77	15450
1983	15497	15560	15500	15540.62	15376	15326.61	15498.37	15533.19	15450
1984	15145	15510	15500	15540.62	15376	15326.61	15060.27	15533.19	15450
1985	15163	15510	15500	15541.27	15287	15268	15411.79	15532.34	15600
1986	15984	15510	15500	15541.27	15287	15268	15698.78	15532.34	15600
1987	16859	16860	17000	16254.50	16523	16854.26	17452.72	16298.77	16950
1988	18150	17060	17000	17040.41	17519	17193.42	17452.72	17113.79	17150
1989	18970	18960	19000	18902.30	19500	19044.67	18964.01	18741.35	19050
1990	19328	19260	19500	19357.30	19000	19337.7	18883.1	19190.44	19350
1991	19337	19110	19250	19168.56	19500	19135.72	18909.87	18972.15	19050
1992	18876	19110	19250	19168.56	19500	19135.72	18717.68	18972.15	19050

TABLE 7. Comparison of accuracy evaluation of the forecasted values of the enrollment data of University of Alabama from 1971 to 1992 by using the proposed SVNHFTS model and other FTS models.

No	Accuracy evaluation	Proposed Algorithm	[40]	[41]	[43]	[44]	[45]	[46]	[48]	Acceptable Range
1	RMSE	342.4142	363.3401	428.6384	493.5640	359.1218	**319.3640**	431.6441	342.6880	---
2	R	0.9790	0.9774	0.9668	0.9594	0.9775	**0.9821**	0.9674	0.9790	$-1 \leq R \leq 1$
3	R^2	0.9583	0.9554	0.9346	0.9205	0.9555	**0.9646**	0.9359	0.9584	$0 \leq R^2 \leq 1$
4	AFE	**1.33%**	1.57%	1.94%	2.34%	1.45%	1.50%	2.04%	1.45%	---
5	PP	0.7957	0.7832	0.7442	0.7055	0.7857	**0.814**	0.7424	0.7955	$PP > 0$
6	MAD	**220.71**	257.381	318.691	386.2381	238.1843	252.009	335.7829	238.9048	---
7	RSFE	-257	-217	**-68.43**	573	-931.25	-645.04	-363.72	433	---
8	TS	-1.1644	-0.8431	**-0.2147**	1.4835	-3.9098	-2.5596	-1.0832	1.8124	$-4 \leq TS \leq 4$

A. ENROLLMENT DATA OF UNIVERSITY OF ALABAMA FROM 1971 TO 1992

The enrollment data of University of Alabama from 1971 to 1992 has been used to test the efficiency and the accuracy of almost any novel fuzzy time series forecasting method since fuzzy time series was proposed by Song & Chissom in 1993 [52]. We also utilize this dataset to test our proposed SVNHFTS forecasting model. The result of the forecasted values is shown in Table 6 and Figure 2, together with the forecasted results from the other FTS models based on improved version of fuzzy sets. The accuracy evaluation of the forecasted values using the proposed SVNHFTS model and other FTS models is shown in Table 7, complete with the ranking in Table 8. Note that the result obtained from [40], [41], [43]–[46], [48] are directly based on their calculation published in their journals for interested readers who want to study their proposed models further.

1) DISCUSSION OF RESULTS

Looking at RMSE, the proposed algorithm does not have the smallest RMSE out of all the models. But since RMSE evaluates the root mean square error of forecasted value to the actual value, the depicted value only considers the exact difference between the forecasted value to the actual value, which can be misleading if one of the forecasted value has an error which is big in numbers but small in terms of relativeness. To double check, we will look at AFE (average forecasting error), since AFE calculates the mean absolute forecasting

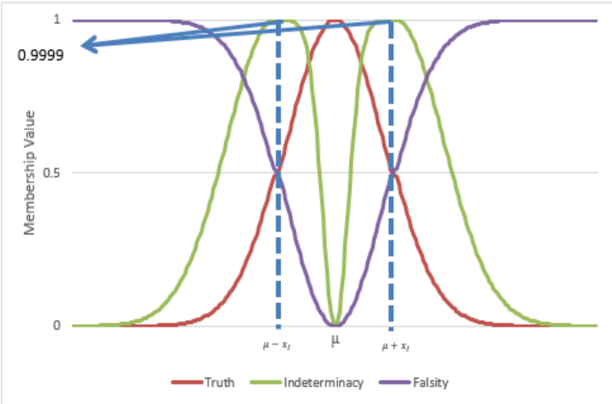

FIGURE 1. Truth, indeterminacy, and falsity membership function of Gaussian function for proposed algorithm of FTS based on SVNHFS.

TABLE 8. Ranking of accuracy evaluation of the forecasted values of the enrollment data of University of Alabama from 1971 to 1992 by using the proposed SVNHFTS model and other FTS models.

No	Accuracy evaluation	Proposed Algorithm	[40]	[41]	[43]	[44]	[45]	[46]	[48]
1	RMSE	2	5	6	8	4	**1**	7	3
2	R	3	5	7	8	4	**1**	6	2
3	R^2	3	5	7	8	4	**1**	6	2
4	AFE	**1**	5	6	8	3	4	7	2
5	PP	2	5	6	8	4	**1**	7	3
6	MAD	**1**	5	6	8	2	4	7	3
7	RSFE	3	2	**1**	6	8	7	4	5
8	TS	4	2	**1**	5	8	7	3	6
	Average	**2.375**	4.25	5	7.375	4.625	3.25	6.875	3.25

error, which is calculated based on the relativeness of the error to the actual value. Based on AFE, the proposed algorithm has the smallest AFE out of all the models, at 1.33%. This means that in terms of the relativeness of the error to the actual value, the proposed algorithm has the best accuracy.

The value of R and R^2 indicates the correlation and the linear association of the forecasted value to the actual value [41]. The value of R and R^2 of the proposed algorithm is not the highest out of all the models, but they are already satisfactory enough at above 0.95 to know that the forecasted value is well associated with the actual value. These values are still better than some of the other models and on par with the range of the values obtained from the other models.

The value of PP indicates the efficiency of the model [59]. The value of PP of the proposed algorithm is not the highest, however it is still on par with the other models, and has a value of almost 80%, indicating that the efficiency of the proposed algorithm is sufficiently satisfactory.

The value of MAD indicates the average of the absolute difference between the forecasted value and the actual value [60]. At 220.71, the MAD of the proposed algorithm is the lowest out of all the compared models, which means that the absolute difference between the forecasted value and the actual value of the model is the lowest out of all the other models.

The value of RSFE indicates the total of the difference between the forecasted value and the actual value, which

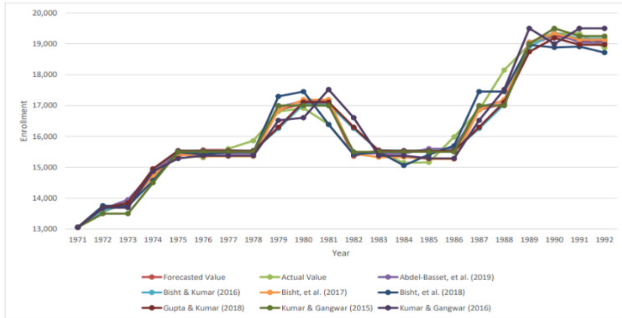

FIGURE 2. Comparison of the proposed SVNHFTS model vs other established FTS models for forecasted values of the enrollment data of University of Alabama from 1971 to 1992.

TABLE 9. Parameters used in each FTS models to forecast yearly closing index of IDX Composite from 1997 to 2018.

Year	Name	Reference	Universe of Discourse	Number of Partition	Length (equal part)	Other Parameter
2015	Kumar & Gangwar	[40]	[350, 6150]	29	200	
2016	Bisht & Kumar	[41]	[−1569, 8055]	29	332	
2016	Kumar & Gangwar	[43]	[350, 6150]	29	200	
2017	Bisht, et al.	[44]	[350, 6150]	29	200	
2018	Bisht, et al.	[45]	[400, 6294]	22	below mean = 148, above mean = 379	
2018	Gupta & Kumar	[46]	[−1569, 8055]	29	332	
2019	Abdel-Basset, et al.	[48]	[350, 6150]	29	200	Membership values of truth, indeterminacy, falsity = [0.9, 0.1, 0.1]
2019	Proposed SVNHFTS	N/A	[350, 6150]	29	200	

TABLE 10. Comparison of forecasted values of yearly average of daily closing index of IDX Composite from 1997 to 2018 by using the proposed SVNHFTS model and other models.

Year	Actual	Proposed Algorithm	[40]	[41]	[43]	[44]	[45]	[46]	[48]
1997	542.6416	---	---	---	---	---	---	---	---
1998	417.0718	547.1429	550	1,096	665	701	641	1,465	650
1999	547.5808	547.1429	550	1,075	665	701	641	1,449	650
2000	507.3768	547.1429	550	1,096	665	701	641	1,465	650
2001	399.8479	547.1429	550	1,075	665	701	641	1,449	650
2002	452.5443	547.1429	550	1,075	665	701	641	1,449	650
2003	508.1458	547.1429	550	1,075	665	701	641	1,449	650
2004	794.9348	547.1429	550	1,075	665	701	641	1,449	650
2005	1089.6011	1090	1,150	1081	1,150	1,166	1,140	1,297	1150
2006	1422.9660	1490	1,350	1601	1,444	1,328	1,436	1,673	1350
2007	2167.2630	2090	2,150	1533	2,150	2,100	2,176	1,583	2150
2008	2101.3375	2423.3333	2,150	2190	2,082	2,283	2,056	2,023	2050
2009	1982.1217	2423.3333	2,150	2190	2,082	2,283	2,159	2,023	2050
2010	3057.0466	2423.3333	3,150	2,190	3,150	2,676	2,159	1,583	3150
2011	3727.3004	3690	3,750	3,842	3,750	3,721	3,641	3,973	3750
2012	4103.0911	4090	4,150	4,724	4,150	4,145	4,020	4,849	4150
2013	4591.7619	4690	4,550	5,437	4,550	4,572	4,711	5,485	4550
2014	4908.7871	4890	4,950	5,178	4,950	5,117	4,915	5,215	4950
2015	4908.1953	4990	5,350	5,384	4,982	5,280	4,915	5,355	5350
2016	5030.2782	4990	5,350	5,384	4,982	5,280	4,915	5,355	5350
2017	5739.6210	5690	5,350	5,478	5,750	5,280	5,888	5,500	5350
2018	6086.7387	6090	6,150	5,399	---	6,084	5,888	5,422	6150

indicates the tendency of the nature, i.e., whether the model tends to over-forecast or under-forecast the values [41]. The value of the RSFE of the proposed algorithm is negative, which means that there is a tendency of the proposed algorithm to under-forecast the forecasted values. The value is also not small compared to some of the other models, which have values closer to 0.

TABLE 11. Comparison of accuracy evaluation of the forecasted values of yearly average of daily closing index of IDX Composite from 1997 to 2018 by using the proposed SVNHFTS model and other models.

No	Accuracy evaluation	Proposed Algorithm	[40]	[41]	[43]	[44]	[45]	[46]	[48]	Acceptable Range
1	RMSE	200.5735	171.9801	518.2221	**121.4441**	227.49	234.83	729.298	184.1728	---
2	R	0.9948	0.9965	0.9715	**0.9989**	0.9944	0.9931	0.9449	0.9963	$-1 \le R \le 1$
3	R^2	0.9897	0.9930	0.9439	**0.9977**	0.9888	0.9863	0.8929	0.9926	$0 \le R^2 \le 1$
4	AFE	9.98%	**9.09%**	47.05%	14.78%	18.77%	14.59%	73.95%	14.38%	---
5	PP	0.8980	0.9125	0.7365	**0.9343**	0.8843	0.8806	0.6291	0.9063	$PP > 0$
6	MAD	123	120.93	454.71	**94.222**	187.2	148.77	621.22	140.1	---
7	RSFE	346.39	1006.4	4632.6	1372	1679.7	**-36.27**	6964.6	1506.4	---
8	TS	2.8162	8.3222	10.1882	14.5615	8.9728	**-0.2438**	11.2111	10.7520	$-4 \le TS \le 4$

TABLE 12. Ranking of the accuracy evaluation of the forecasted values of yearly average of IDX Composite from 1997 to 2018 by using the proposed SVNHFTS model and other models.

No	Accuracy evaluation	Proposed Algorithm	[40]	[41]	[43]	[44]	[45]	[46]	[48]
1	RMSE	4	2	7	**1**	5	6	8	3
2	R	4	2	7	**1**	5	6	8	3
3	R^2	4	2	7	**1**	5	6	8	3
4	AFE	2	**1**	7	5	6	4	8	3
5	PP	4	2	7	**1**	5	6	8	3
6	MAD	3	2	7	**1**	6	5	8	4
7	RSFE	2	3	7	6	5	**1**	8	4
8	TS	2	3	5	8	4	**1**	7	6
	Average	3.125	**2.125**	6.75	3	5.125	4.375	7.875	3.625

TABLE 13. Parameters used in each FTS models to forecast weekly closing index of MERVAL Index from February 15th, 2019 to September 20th, 2019.

Year	Name	Reference	Universe of Discourse	Number of Partition	Length (equal part)	Other Parameter
2015	Kumar & Gangwar	[40]	[24500, 44500]	20	1000	
2016	Bisht & Kumar	[41]	[19599, 49365]	20	2977	
2016	Kumar & Gangwar	[43]	[24500, 44500]	20	1000	
2017	Bisht, et al.	[44]	[24500, 44500]	20	1000	
2018	Bisht, et al.	[45]	[24609, 44908]	31	below mean = 519, above mean = 773	
2018	Gupta & Kumar	[46]	[19599, 49365]	20	2977	
2019	Abdel-Basset, et al.	[48]	[24500, 44500]	20	1000	Membership values of truth, indeterminacy, falsity = [0.9, 0.1, 0.1]
2019	Proposed SVNHFTS	N/A	[24500, 44500]	20	1000	

The value of TS indicates the biasness and the tendency of under or over-forecast. The satisfactory range for TS is between −4 and 4 [41]. Since the value of TS of the proposed algorithm is at −1.1644, this indicates that the proposed algorithm is not overly biased and does not produce extreme over or under forecasted values, compared to some of the other models which have TS with a value closer to the limit of the permissible satisfactory range of between −4 and 4.

B. IDX COMPOSITE (INDONESIA'S MARKET INDEX) YEARLY AVERAGE OF DAILY CLOSING INDEX FROM 1997 TO 2018

To further demonstrate the performance of SVNHFTS model, we also use the market index of Indonesia, IDX Composite. The data used from IDX Composite is from 1997 to 2018, and the prediction is done on a yearly basis, with the actual data for each year is calculated as the average of the daily closing index in the respective year. This set of time series data is unique that the first eight data are relatively stationary compared to the rest of the time series data, and we want to test how our proposed model performs with the added stationary factor compared to the other seven FTS models [40], [41], [43]–[46], [48].

TABLE 14. Comparison of forecasted values of weekly closing index of MERVAL Index from February 15th, 2019 to September 20th, 2019 by using the proposed SVNHFTS model and other FTS models.

Date	Actual	Proposed Algorithm	[40]	[41]	[43]	[44]	[45]	[46]	[48]
15/2/2019	37469.97	---	---	---	---	---	---	---	---
22/2/2019	36646.79	35700	36500	36417.93	36500	36210.60	36404.89	37732.55	36500
1/3/2019	33834.86	35700	33500	38280.39	33500	35099.38	33950.56	39502.21	33500
8/3/2019	33020.07	34200	33500	34670.25	33515.02	34005.63	34005.26	35573.29	33833.33
15/3/2019	33933.08	33366.67	33500	32961.89	32006.41	34005.63	32852.64	34337.74	33833.33
22/3/2019	32827.98	34200	33500	34670.25	33515.02	34005.63	34005.26	35573.29	33833.33
29/3/2019	33466.03	33366.67	32000	33588.92	32006.41	32332.63	32852.64	34337.74	31833.33
5/4/2019	32666.88	33366.67	33500	33588.92	33515.02	34005.63	34005.26	34337.74	33833.33
12/4/2019	31357.03	33366.67	32000	33588.92	32006.41	32332.63	32852.64	34337.74	31833.33
17/4/2019	32036.78	32200	32500	32961.89	32545.42	31614.37	31874.56	32661.63	32500
26/4/2019	30001.10	31700	32000	32961.89	32006.41	32332.63	29942.52	31061.88	31833.33
3/5/2019	32412.27	29533.33	28500	30263.21	32579.35	30219.89	32393.56	31061.88	29833.33
10/5/2019	33393.81	31700	32000	32961.89	32006.41	32332.63	33431.56	32661.63	31833.33
17/5/2019	33315.70	33366.67	33500	33588.92	33515.02	34005.63	34005.26	34337.74	33833.33
24/5/2019	35084.92	33366.67	33500	33588.92	33515.02	34005.63	34005.26	34337.74	33833.33
31/5/2019	33949.53	34200	37000	36417.93	33500	35726.36	37373.05	37732.55	37000
7/6/2019	35662.87	34200	33500	34670.25	33515.02	34005.63	34005.26	35573.29	33833.33
14/6/2019	40487.61	40200	37000	38280.39	40500	35726.36	37373.05	38295.55	37000
21/6/2019	40294.82	41533.33	40500	40350.17	41328	41392.22	36404.89	39502.21	41000
28/6/2019	41796.36	41533.33	40500	40350.17	41328	41392.22	36404.89	39502.21	41000
5/7/2019	41755.69	42200	43000	39670	42489.01	42175.93	41967.12	39244.08	42833.33
12/7/2019	42753.10	42200	43000	39670	42489.01	42175.93	41967.12	39244.08	42833.33
19/7/2019	40161.60	40200	40500	37963.50	40876.50	40440.94	40269.89	37369.54	40500
26/7/2019	41983.74	41533.33	40500	40350.17	41328	41392.22	36404.89	39502.21	41000
2/8/2019	41359.15	42200	43000	39670	40876.50	42175.93	41967.12	39244.08	42833.33
9/8/2019	44355.09	44200	43000	40655.53	42489.01	42175.93	44134.89	40185.50	42833.33
16/8/2019	30406.65	30200	30500	35025.60	30500	32519.86	30317.56	34898.85	30500
23/8/2019	26585.97	29533.33	28500	30263.21	29624	30219.89	28389.29	31061.88	29833.33
30/8/2019	24608.56	25200	25500	28177.86	---	28452.17	---	28129.49	25500
6/9/2019	27659.66	28200	28000	28035.45	---	27576.27	27722.56	28051.96	27500
13/9/2019	30136.28	30200	30500	28924.54	30500	29776.85	30317.56	28789.62	30500
20/9/2019	30060.46	29533.33	28500	30263.21	29624	30219.89	28389.29	29071.45	29833.33

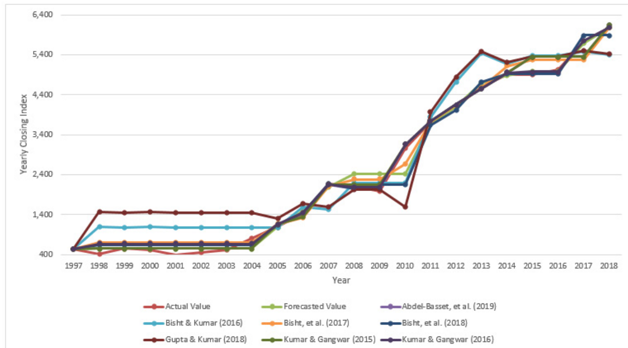

FIGURE 3. Comparison of the proposed SVNHFTS model vs other established FTS models for forecasted value of yearly average of daily closing index of IDX Composite from 1997 to 2018.

The prediction is done by using the proposed SVNHFTS model and the other seven FTS models [40], [41], [43]–[46], [48] listed in Table 3, with the parameter for each proposed model listed in Table 9, and the result, the accuracy evaluation, and the ranking of the accuracy evaluation listed in Table 10, Table 11, and Table 12, respectively. The comparison of all of the forecasted values to the other FTS models can be seen in Figure 3.

1) DISCUSSION OF RESULTS

Compared to the enrollment data of University of Alabama, the yearly closing index of IDX Composite from 1997 to 2018 has wider range of values and more drastic movement, whereas the data from 1997 to 2003 has smaller range of values and relatively stationary compared to the other years. This is the main reason that the AFE for all of the models compared is bigger than 9%. The proposed SVNHFTS model has AFE of 9.98%, which is the second best from all of the compared models, with the other models' AFE is mostly more than 14%. This can be contributed to the fact that SVNHFTS model takes into account the repetition of the NLRs during

TABLE 15. **Comparison of accuracy evaluation of the forecasted values of weekly closing index of MERVAL Index from February 15th, 2019 to September 20th, 2019 by using the proposed SVNHFTS model and other FTS models.**

No	Accuracy evaluation	Proposed Algorithm	[40]	[41]	[43]	[44]	[45]	[46]	[48]	Acceptable Range
1	RMSE	1198.31	1521.23	2204.46	**1137.13**	1706.43	1969.32	2542.78	1445.86	---
2	R	**0.9720**	0.9545	0.9169	0.9704	0.9451	0.9177	0.88	0.9585	$-1 \leq R \leq 1$
3	R^2	**0.9448**	0.9111	0.8407	0.9417	0.8932	0.8421	0.7743	0.9187	$0 \leq R^2 \leq 1$
4	AFE	2.74%	3.41%	5.35%	**2.56%**	3.92%	3.46%	6.20%	3.24%	---
5	PP	**0.7635**	0.6998	0.565	0.7564	0.6633	0.5892	0.4982	0.7147	$PP > 0$
6	MAD	896.92	1168.4	1802.1	**867.06**	1287.6	1263.1	2111.8	1103.5	---
7	RSFE	4185.56	-5014	4817.8	-2047	6038.7	-13416	9238.9	**985.56**	---
8	TS	4.6666	-4.2916	2.6734	-2.3610	4.69	-10.6214	4.375	**0.8932**	$-4 \leq TS \leq 4$

TABLE 16. **Ranking of the accuracy evaluation of the forecasted values of weekly closing index of MERVAL Index.**

No	Accuracy evaluation	Proposed Algorithm	[40]	[41]	[43]	[44]	[45]	[46]	[48]
1	RMSE	2	4	7	**1**	5	6	8	3
2	R	**1**	4	7	2	5	6	8	3
3	R^2	**1**	4	7	2	5	6	8	3
4	AFE	2	4	7	**1**	6	5	8	3
5	PP	**1**	4	7	2	5	6	8	3
6	MAD	2	4	7	**1**	6	5	8	3
7	RSFE	3	5	4	2	6	8	7	**1**
8	TS	6	4	3	2	7	8	5	**1**
	Average	2.25	4.125	6.125	**1.625**	5.625	6.25	7.5	2.5

the de-neutrosophication process. This result means that the proposed SVNHFTS model is quite satisfactory in handling data with a wide range of movement compared to the other established FTS models.

The value of R and R^2 of the proposed SVNHFTS model is not the best out of all the compared FTS models. However, with the value of around 0.99, the value of R and R^2 are on par with the other models, and this also means that the predicted and actual values of the yearly closing index is almost perfectly correlated.

The value of PP of the proposed SVNHFTS model is 0.8980. Even though this value is not the best out of all the other compared FTS models, the value is already on par with the other models and this means that SVNHFTS model is satisfactorily efficient in getting the predicted values. Note that for [43], there is no available prediction value in 2018, which makes the value of the RMSE smaller compared to the other models.

The value of MAD and RSFE of the proposed SVNHFTS model are 123 and 346.39, respectively, which results in a TS of 2.8162, which is still within the acceptable range. Almost all of the other compared FTS models with exception to [45] fail to achieve a TS value between −4 and 4. This means that SVNHFTS model is quite satisfactory in not over-forecasting or under-forecasting the predicted values.

C. WEEKLY CLOSING INDEX OF MERVAL INDEX FROM FEBRUARY 15 th, 2019 TO SEPTEMBER 20th, 2019

For the final dataset, we choose to use MERVAL Index, which is Argentinian market index. At the time of writing, MERVAL Index had just suffered a steep decline in August 2019, which was the worst decline of any stock in the world recorded by Bloomberg since 1950 [61]. This caused quite a really volatile data, and we want to test whether our proposed SVNHFTS model's performance is still adequate in such a volatile dataset compared to the other FTS models [40], [41], [43]–[46], [48]. We use the weekly closing index from six months before the steep decline, starting from February 15th, 2019 to September 20th, 2019, which is one month after the decline.

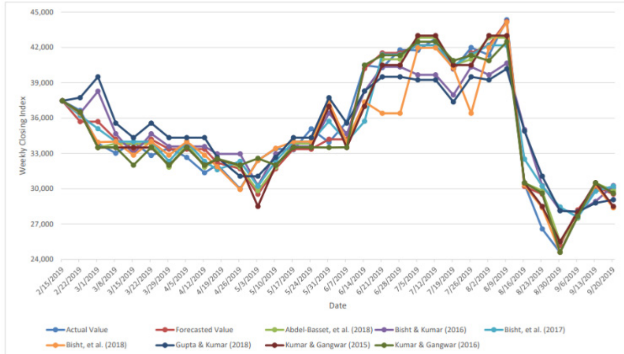

FIGURE 4. **Comparison of the proposed SVNHFTS model vs other established FTS models for forecasted value of weekly closing index of MERVAL Index from February 15th, 2019 to September 20th, 2019.**

Same like IDX Composite, the prediction is done by using the proposed SVNHFTS model and the other seven FTS models [40], [41], [43]–[46], [48] listed in Table 3, with the parameter for each proposed model listed in Table 13, and the result, the accuracy evaluation, and the ranking of the accuracy evaluation listed in Table 14, Table 15, and Table 16, respectively. The comparison of all of the forecasted values to the other FTS models can be seen in Figure 4.

1) DISCUSSION OF RESULTS

With the more volatile dataset and a wider range of values compared to the dataset in IDX Composite, the forecast accuracy of MERVAL index for the proposed SVNHFTS model and all of the other tested FTS models is generally significantly better compared to the forecast accuracy of IDX Composite, at around 3%.

The proposed SVNHFTS model has RMSE and AFE of 1198.31 and 2.74%, which is the second best from all of the compared FTS models. However, looking at the best RMSE and AFE from [43], these values do not take into account two of the dates at August 30th, 2019 and September 6th, 2019 that are left blank because of no available forecasted values. This means that our proposed SVNHFTS model has the best result out of all the other FTS models with complete forecasted values. Furthermore, the value of R, R^2, and PP of the proposed SVNHFTS model is the best out of all the compared FTS models, at 0.9720, 0.9448, and 0.7635, respectively. This also means that the predicted and actual values of the weekly closing index is almost perfectly correlated, with the best efficiency out of all the compared FTS models.

The value of MAD and RSFE of the proposed SVNHFTS model are 896.92 and 4185.56, respectively, which results in a TS of 4.6666, which is already outside of the acceptable range, even though the value of the MAD is the second best out of all the other compared FTS models. This means that with a more volatile dataset, SVNHFTS model can have a tendency to over-forecast the values, which needs to be taken into account during forecasting.

VI. CONCLUSION AND REMARKS

In this paper, we have proposed a novel first-order single-valued neutrosophic hesitant fuzzy time series forecasting model, improving the previously proposed neutrosophic time series [73] by:

(i) Incorporating the degree of hesitancy by using single-valued neutrosophic hesitant fuzzy set [51] instead of single-valued neutrosophic set [35] to better capture the uncertainty and fuzziness of the data movement
(ii) Adding an algorithm that automatically converts the crisp dataset into neutrosophic set in terms of truth, falsity, and indeterminacy membership function [55] that eliminates the need of expert's opinion to give the weightage of truth, falsity, and indeterimancy membership function in each of the partitioned neutrosophic set
(iii) Incorporating Markov Chain algorithm [19] in the deneutrosophication process to include the weightage of repeating NLRs.

We have also established the advantage of our proposed SVNHFTS model to the other FTS models from the improved versions of fuzzy sets, which are:

(i) Incorporating degree of hesitancy to better capture the uncertainty of the data movement, which results in better forecasting accuracy
(ii) Simplified deneutrosophication process by eliminating the use of max-min operator to reduce time and energy consumptions.
(iii) Guaranteed forecast values in each of the time interval
(iv) Taking into account the weightage of repeated NLRs.

We have also applied our proposed SVNHFTS model to real datasets while also comparing the result to the other FTS models with improved versions of fuzzy set, and the results of our finding are:

(i) Using the benchmark enrollment data of University of Alabama from 1971 to 1992, our proposed model has the best AFE and MAD out of the other FTS models.
(ii) Using the yearly average of daily closing index of IDX Composite from 1997 to 2018, which has a relatively stationary interval in the first seven data, although the predicted value's result from the proposed model is not the best out of the other models, the performance of the proposed model is sufficiently satisfactory, with the values of the evaluation parameters on par with the other established FTS models and within the acceptable range
(iii) Using the weekly closing index of MERVAL index from February 15th, 2019 to September 20th, 2019, which is quite volatile, the performance of the proposed model is the best out of all the other FTS models in RMSE, AFE, R, R^2, and PP out of all the other FTS models with guaranteed forecasted result. However, the value of TS is outside of the acceptable range, which can indicate the tendency of our proposed SVNHFTS model to over-forecast in a volatile dataset.

Our suggestions for the direction of the possible future research based on our findings, which are:

(i) Developing an improved SVNHFTS model with higher-order NLRs.
(ii) Utilizing the SVNHFTS model to predict other types of dataset and evaluating the accuracy of the model.
(iii) Improving the algorithm of the proposed SVNHFTS model to get better accuracy for the predicted values, e.g. using improved partition length optimization method to partition the universe of discourse, reviewing the neutrosophication and de-neutrosophication rule and developing improved algorithm.

To conclude our paper, we provide our suggestions for the direction of the future directions of development of NTS models based on improved version of fuzzy sets:

(i) Applying the model to other real-life situations, comparing the result with the existing time-series models that has been applied widely for these kinds of situation.
(ii) Integrating the model into the widely used existing time-series models, analyzing the predictive capabilities and the accuracy of the model.
(iii) Utilizing a more sophisticated neutrosophic set to develop new NTS model, e.g. probabilistic single-valued neutrosophic hesitant fuzzy set [62].

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CONFLICTS OF INTEREST

The authors declare that there is no conflict of interest

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